Released Form

Student Name: _____

Spring 2013 North Carolina Measures of Student Learning: NC's Common Exams **Precalculus**

Student Booklet

READY

Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314

1 What are the ***approximate*** rectangular coordinates for the point with polar coordinates (5, 30°)?

A (2.5, 2.89)

B (2.5, 4.33)

C (2.89, 4.33)

D (4.33, 2.5)

$x = r\cos\theta = 5(.867) \approx 4.33$

$y = r\sin\theta = 5(\frac{1}{2}) = 2.5$

2 A sequence is shown below.

1 2 3 4 5

6, 12, 20, 30, 42, 56, . . .

Which is the recursive formula for this sequence?

A $t_n = n + 2(t_{n-1} + 1)$

B $t_n = (t_{n-1} + 1)(n - 2)$

C $t_n = 2(t_{n-1} + 2) - (n + 2)$

D $t_n = t_{n-1} + 2(n + 1)$

$t_2 = 6 + 2(2+1) = 12$

$t_3 = 12 + 2(3+1) = 20$

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3 A quadratic function, f, has zeros P and Q, such that $P + Q = 5$ and $\frac{1}{P} + \frac{1}{Q} = 8$.

Which choice describes f?

A $f(x) = 8x^2 - 40x + 5$

B $f(x) = 8x^2 - 40x - 5$

C $f(x) = 2x^2 - 10x + 5$

D $f(x) = 2x^2 - 10x - 5$

4 Lucy invested \$6,000 into an account that earns 6% interest compounded continuously. ***Approximately*** how long will it take for Lucy's investment to be valued at \$25,000?

A 52.7 years

B 46.9 years

C 24.5 years

D 23.8 years

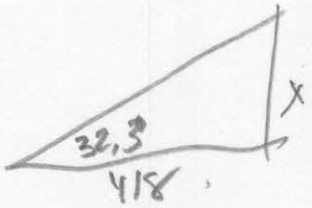
5 A lamppost is located 418 feet from a building. The angle of elevation from the base of the lamppost to the top of the building is 32.3°. ***Approximately*** how tall is the building?

A 223 feet

B 264 feet

C 510 feet

D 661 feet



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6 Two functions are shown below.

$$T(x) = -x$$
$$P(x) = 10x + 2$$

What is the value of $P(T(3)) - T(P(3))$?

A 8

B 4

C 0

D -4

$P(-3) - T(32)$

$(-30+2) - (-32)$

$-30+2+32 = 4$

7 A piecewise function is shown below.

$$f(x) = \begin{cases} cx + 1, & x \leq 2 \\ cx^2 - 1, & x > 2 \end{cases}$$

For what value of c does $\lim_{x \to 2} f(x)$ exist?

A -2

B -1

C 1

D 4

$cx+1 = cx^2-1$

$2c+1 = 4c-1$

$2c = 2$

$c = 1$

Go to the next page.

8 What are the polar coordinates of (4, 9)?

A $(\sqrt{97}, 66°)$

B $(\sqrt{97}, 114°)$

C $(\sqrt{13}, 66°)$

D $(\sqrt{13}, 114°)$

$x = r\cos\theta = 4$

$y = r\sin\theta = 9$

$r^2(1) = 16 + 81 = 97$

$r = \sqrt{97}$

$\tan\theta = \frac{y}{x} = \frac{9}{4} = 2.25$

$\theta = \tan^{-1}(2.25)$

$= 66°$

9 A sequence is shown below.

$1, 3, 3^2, 3^3, \ldots$

How many terms of the sequence must be added together for the sum to equal 3,280?

A 6

B 7

C 8

D 9

Eq. Sheet $S = a_1\frac{(1-r^n)}{1-r} = 3280$

$1 - r^n = 3280 - 3280r$

$r = 3$

$1 - 3^n = 3280 - 9840$

$1 - 3^n = -6560$

$3^n = 1 + 6560$

$\log_3 3^n = \log_3 6561$

$n = 8$

$\frac{1-3^n}{(-2)} - 3280 = 0$

OR Graph

Go to the next page.

10 The first term of an infinite geometric sequence is 2. The sum of the sequence is 6. What is the common ratio of the sequence?

A $\frac{1}{3}$

B $\frac{2}{3}$

C $\frac{3}{3}$

D $\frac{4}{3}$

$S = 2\left(\frac{1}{1-r}\right) = 6$

$\frac{1}{1-r} = 3$

$1 = 3 - 3r$

$3r = 2$

$r = \frac{2}{3}$

11 Which is true of the series shown below?

$$\pi + \frac{3\pi}{4} + \frac{9\pi}{16} + \frac{27\pi}{64} + \ldots$$

A The series diverges.

B The series converges to $\frac{3\pi}{2}$.

C The series converges to $\frac{4\pi}{3}$.

D The series converges to 4π.

$r = \frac{3}{4}$

$S = \pi\left(\frac{1}{1-\frac{3}{4}}\right) = 4\pi$

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12 Karen recursively generated a sequence of five positive integers by starting with a positive integer, a_1, and then applying the recursive formula $a_n = a_{n-1} + 3n - 1$ to generate a_n for $n = 2, 3, 4$, and 5.

If the value of a_5 was 407, what was the value of Karen's starting term, a_1?

A 366 $a_1 = 366$ $a_2 = 366 + 6 - 1 = 371$

B 367 $a_3 = 371 + 9 - 1 = 379$ $a_5 = a_4 + 3(5) - 1$

C 368 $a_4 = 379 + 12 - 1 = 390$ $a_4 = a_5 + 14$

D 369 $a_5 = 390 + 15 - 1 = 404$

393 a_4

$a_2 = a_1 + 3(2) - 1$

$a_1 =$

$369 + (3 \cdot 2) - 1 = 374$

$374 + (3 \cdot 3) - 1 = 382$

$382 + 3 \cdot 4 - 1 = 393$

$393 + (3 \cdot 5) - 1$

407 ✓

13 What is the distance between y-intercepts of the graph of $x + 8 = 2(y + 3)^2$?

A 4

B 6

C 11

D 15

$x = 0$

$8 = 2(y + 3)^2$

$4 = (y + 3)^2$

$\pm 2 = (y + 3)$

$y = -3 \pm 2$

means 4

14 Which is a solution set to $x + \frac{3x}{x - 1} = \frac{x + 2}{x - 1}$?

A $\{-1\}$

B $\{-2\}$

C $\{-2, 1\}$

D $\{2, -1\}$

$x(x - 1) + 3x = x + 2$

$x^2 - x + 3x = x + 2$

$x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0$

Go to the next page.

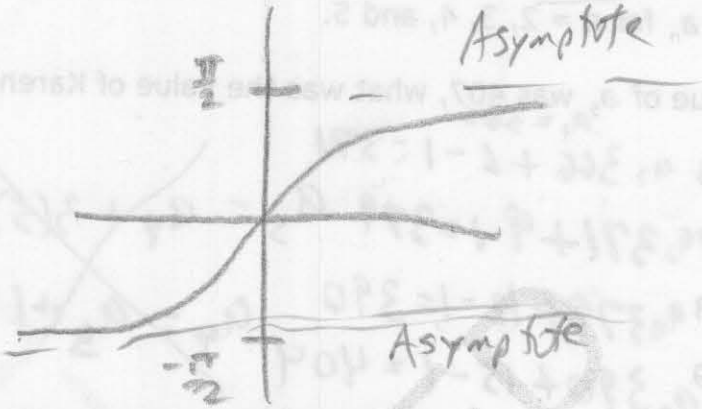
15 What is the range of the inverse of $y = \tan x$?

A $-\frac{\pi}{2} < y < \frac{\pi}{2}$

B $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

C $0 < y < \pi$

D $0 \leq y \leq \pi$



16 James is standing 10 meters away from Samantha.

- A bird is located in the sky at a point between where James and Samantha are standing.
- James is looking up at the bird at an angle of elevation of 74°.
- Samantha is looking up at the bird at an angle of elevation of 47°.

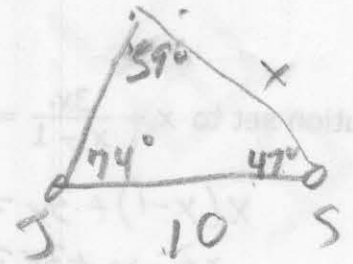
Approximately how far is the bird from Samantha?

A 7.6 meters

B 8.5 meters

C 11.2 meters

D 13.1 meters



$$\frac{\sin 59^\circ}{10} = \frac{\sin 74^\circ}{x}$$

$$x = 10\,\frac{\sin 74^\circ}{\sin 59^\circ} \approx 11.2$$

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17 What is the inverse function of $f(x) = \log_5(2x - 1)$?

A $f^{-1}(x) = 5^x - 1$

B $f^{-1}(x) = \frac{5^x + 1}{2}$

C $f^{-1}(x) = \log_2(5x - 1)$

D $f^{-1}(x) = \log_5 \frac{5x + 1}{2}$

$y = \log_5(2x-1)$

$x = \log_5(2y-1)$

$5^x = (2y-1)$

$y = \frac{5^x+1}{2}$

18 What is the value of the limit shown below?

$$\lim_{n\to\infty}\left(\frac{3^n - 1}{3^n}\right) = 1$$

A $\frac{1}{3}$

B $\frac{2}{3}$

C 1

D $+\infty$

$\lim_{n\to\infty} \frac{(3^n)}{3^n} = 1$

19 What type of conic section is represented by $r = \frac{8}{16 + 125\sin\theta}$?

A circle

B ellipse

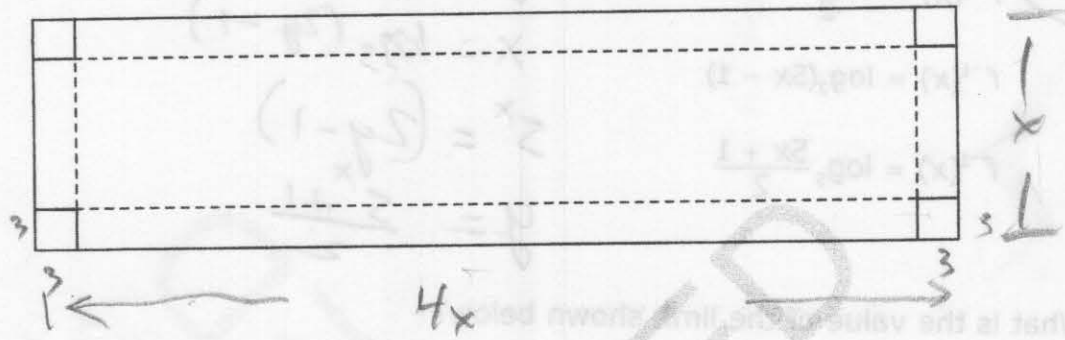
C hyperbola

D parabola

Lots of Zoom

Go to the next page.

20 James had a rectangular piece of cardboard that was four times as long as it was wide. He wanted to use the cardboard to make a box with no lid. To do this, he first cut a 3-by-3-inch square out of each of the four corners of the piece of cardboard, as shown in the picture below.



Then James folded the cardboard along the four dotted lines shown in the picture. This created an open box with a volume of 336 cubic inches.

What was the width of the sheet of cardboard that James started with?

A 10.5 inches

B 9.5 inches

C 8.5 inches

D 7.5 inches

$(4x-6)(x-6)3 = 336$

$4x^2 - 24x - 6x + 36 = 112$

$4x^2 - 30x - 76 = 0$

21 Which expression is equivalent to $(\sec\theta)\left(\frac{\sin\theta}{\tan\theta}\right)$?

A $\cos^2\theta - \sin^2\theta$

B $\sin^2\theta - \cos^2\theta$

C $\cot^2\theta - \csc^2\theta$

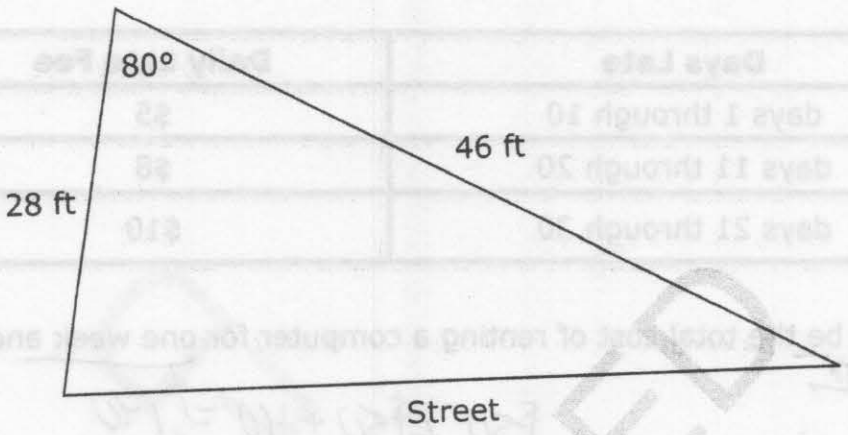
D $\csc^2\theta - \cot^2\theta$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}} = \frac{\sin\theta}{\sin\theta} = 1$

Use ID $\cot^2\theta + 1 = \csc^2\theta$

$1 = \csc^2\theta - \cot^2\theta$

22 Suppose that for each foot of land along the street, the annual tax is $25 per foot. The diagram below shows a plot of land.



About how much is the annual tax for the plot?

Law of Cosines

$c^2 = 28^2 + 46^2 - 2(28)(46)\cos 80°$

$c = 49.52$

$\times 25 = 1238$

A $1,238

B $1,293

C $1,321

D $1,411

23 The function $C(x) = \frac{2.50x + 1.00}{x}$ models the cost per item for a company to produce x items after the first item is made. What is the inverse function of $C(x)$?

A $C^{-1}(x) = \frac{1.00}{x - 2.50}$

B $C^{-1}(x) = \frac{x - 2.50}{1.00}$

C $C^{-1}(x) = \frac{x - 1.00}{2.50}$

D $C^{-1}(x) = \frac{2.50}{x - 1.00}$

$y = \frac{2.50x + 1.00}{x}$

$x = \frac{2.50y + 1}{y}$

$xy - 2.5y = 1$

$y(x - 2.5) = 1$

$y = \frac{1}{x - 2.5}$

Go to the next page.

24 A computer rental company charges $50 to rent a computer for one week. The table below shows the daily late fees the company charges if a computer is returned late.

Days Late	Daily Late Fee
days 1 through 10	$5
days 11 through 20	$8
days 21 through 30	$10

What would be the total cost of renting a computer for one week and returning it 15 days late?

$50 + $50 + 40 = $140

A $120

B $125

C $140

D $170

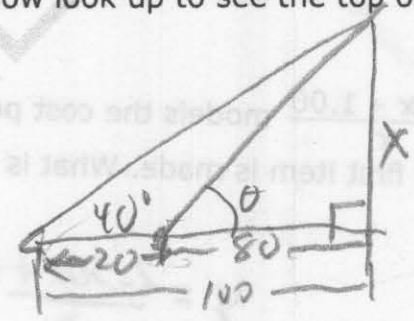
25 From a point 100 feet from the base of a building, Angie looks up at a 40° angle to the top of a building. She walks 20 feet closer to the building. At ***approximately*** what angle must Angie now look up to see the top of the building?

A 32°

B 46°

C 60°

D 77°



$\tan\theta = \frac{x}{80}$

$x = 80\tan\theta$

$\tan 40° = \frac{x}{100}$

$x = 100\tan 40°$

$80\tan\theta = 100\tan 40°$

$\tan\theta = \frac{100}{80}\tan 40°$

$\theta = \tan^{-1}(1.25\tan 40°) = 46.4°$

This is the end of the multiple-choice portion of the test.

Go to the next page.

The questions you read next will require you to answer in writing.

1. **Write your answers on separate paper.**
2. **Be sure to write your name on each page.**

$x^2 = 18y = 4py$ $FW = 18$

1 The equation $y = \frac{1}{18}x^2$ represents the mirror inside a parabolic lamp.

- What is the focal width of the mirror?
- Use the equation to explain your answer.

2 The function $P(t) = 1,440e^{-0.0259t}$ models the number of cars a dealership sold t years after the first year it was open.

- By what percent is the number of cars being sold decreasing each year? -2.59%
- How many cars did the dealership sell the year it opened?

$P(0) = 1440$ cars

3 Two parametric equations are shown below.

$$x = \frac{3t^2}{2}$$

$$y = 4t - 1$$

$t = \frac{y+1}{4}$

$x = \frac{3\left(\frac{y+1}{4}\right)^2}{2}$

$x = \frac{3}{32}(y+1)^2$ $(y+1)^2 = \frac{32}{3}x$

- Convert the parametric equations into rectangular form.
- Determine what type of equation the rectangular form describes.

Parabola opening to the right

vertex $(0, -1)$ $4p = \frac{32}{3}$ $p = \frac{32}{12}$

$x = -\frac{32}{3}$

$F\left(\frac{32}{12}, -1\right)$

Go to the next page.

This is the end of the Precalculus test.

1. **Look back over your answers.**
2. **Put all of your papers inside your test book and close the test book.**
3. **Place your calculator on top of the test book.**
4. **Stay quietly in your seat until your teacher tells you that testing is finished.**